

JEE-MAIN EXAMINATION – JANUARY 2026

(HELD ON WEDNESDAY 21st JANUARY 2026)

TIME : 3:00 PM TO 6:00 PM

MATHEMATICS

SECTION-A

1. The positive integer n , for which the solutions of the equation

$$x(x+2) + (x+2)(x+4) + \dots + (x+2n-2)(x+2n) = \frac{8n}{3}$$

are two consecutive even integers, is :-

- (1) 3 (2) 6
(3) 12 (4) 9

Ans. (1)

Sol. $x(x+2) + (x+2)(x+4) + \dots + (x+2n-2)(x+2n) = \frac{8n}{3}$

$$\Rightarrow \sum_{r=1}^n (x+2r-2)(x+2r) = \frac{8n}{3}$$

$$nx^2 + 2x \sum_{r=1}^n (2r-1) + 4 \sum_{r=1}^n r(r-1) = \frac{8n}{3}$$

$$nx^2 + 2x \cdot n^2 + \frac{4n(n^2-1)}{3} - \frac{8n}{3} = 0$$

$$x^2 + 2nx + \frac{4(n^2-1)}{3} - \frac{8}{3} = 0 \quad \alpha < \beta$$

$$\therefore |\alpha - \beta| = 2 \Rightarrow \frac{\sqrt{D}}{|a|} = 2 \Rightarrow D = 4$$

$$\Rightarrow 4n^2 - 4 \left(4 \frac{(n^2-1)}{3} - \frac{8}{3} \right) = 4$$

$$\Rightarrow n^2 - \frac{4n^2}{3} = -3$$

$$\Rightarrow n^2 = 9$$

$$\Rightarrow n = 3$$

TEST PAPER WITH SOLUTION

2. Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbf{R}$ and $f'(a-1) = 0$, where a is a real number. Let $g(x) = f(\tan^2 x - 2 \tan x + a)$, $0 < x < \frac{\pi}{2}$.

Consider the following two statements :

(I) g is increasing in $\left(0, \frac{\pi}{4}\right)$

(II) g is decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Then,

- (1) Neither (I) nor (II) is True
(2) Only (II) is True
(3) Only (I) is True
(4) Both (I) and (II) are True

Ans. (1)

Sol. $g(x) = f(\tan^2 x - 2 \tan x + a - 1)$
 $g'(x) = f'((\tan x - 1)^2 + a - 1) \cdot 2(\tan x - 1) \sec^2 x$

$$\because f'(a-1) = 0 \text{ and } f''(x) > 0$$

$$\therefore f'((\tan x - 1)^2 + a - 1) > 0$$

$$g'(x) > 0 \text{ if } (\tan x - 1) > 0$$

$$g \text{ is increasing in } x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

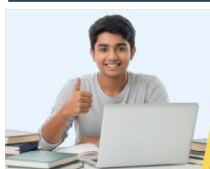
$$g'(x) < 0 \text{ if } \tan x - 1 < 0$$

$$g \text{ is decreasing in } x \in \left(0, \frac{\pi}{4}\right)$$

3. Let $f(x) = x^3 + x^2 f'(1) + 2x f''(2) + f'''(3)$, $x \in \mathbf{R}$. Then the value of $f'(5)$ is :

- (1) $\frac{62}{5}$ (2) $\frac{657}{5}$
(3) $\frac{2}{5}$ (4) $\frac{117}{5}$

Ans. (4)



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Sol. $f'(x) = 3x^2 + 2xf'(1) + 2f'(2)$
 $f'(x) = 6x + 2f'(1)$
 $f'(2) = 12 + 2f'(1)$
 $\therefore f'(x) = 3x^2 + 2xf'(1) + 2(12 + 2f'(1))$
 $f'(x) = 3x^2 + 2(x+2)f'(1) + 24$
 Putting, $x = 1$
 $f'(1) = 3 + 6f'(1) + 24$
 $-5f'(1) = 27 \Rightarrow f'(1) = \frac{-27}{5}$
 $\therefore f'(2) = 12 + 2\left(\frac{-27}{5}\right) = 12 - \frac{54}{5} = \frac{6}{5}$
 $\therefore f'(x) = 3x^2 - \frac{54}{5}x + \frac{12}{5}$
 $\therefore f'(5) = 75 - 54 + \frac{12}{5} = \frac{117}{5}$

4. In the line $\alpha x + 4y = \sqrt{7}$, where $\alpha \in \mathbb{R}$, touches the ellipse $3x^2 + 4y^2 = 1$ at the point P in the first quadrant, then one of the focal distances of P is :

(1) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{11}}$ (2) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{5}}$
 (3) $\frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{5}}$ (4) $\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$

Ans. (4)

Sol. $\alpha x + 4y - \sqrt{7} = 0$ touches $3x^2 + 4y^2 = 1$
 $\therefore c^2 = a^2m^2 + b^2$

$\frac{7}{16} = \frac{1}{3} \times \frac{\alpha^2}{16} + \frac{1}{4} \Rightarrow \alpha = 3, -3$

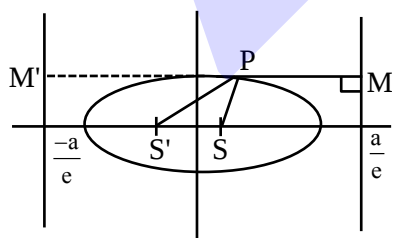
Tangent is $3x + 4y - \sqrt{7} = 0$

Let the point of contact is $P(x_1, y_1)$

$\therefore$ Tangent is $3xx_1 + 4yy_1 = 1$

$\therefore \frac{3x_1}{3} = \frac{4y_1}{4} = \frac{1}{\sqrt{7}} \therefore P\left(\frac{1}{\sqrt{7}}, \frac{1}{\sqrt{7}}\right)$

$e = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$



$PS = e(PM)$

$= e \left(\frac{a}{e} - \frac{1}{\sqrt{7}} \right)$

$= \frac{1}{2} \left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{7}} \right) = \frac{1}{\sqrt{3}} - \frac{1}{2\sqrt{7}}$

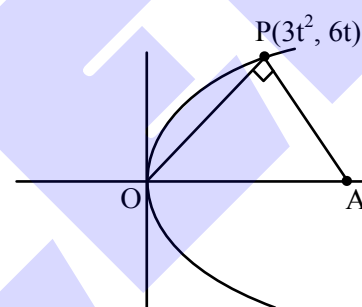
$PS' = e(PM') = \frac{1}{2} \left(\frac{a}{e} + \frac{1}{\sqrt{7}} \right) = \frac{1}{2} \left(\frac{1}{\sqrt{7}} + \frac{2}{\sqrt{3}} \right)$
 $= \frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{7}}$

5. Let $y^2 = 12x$ be the parabola with its vertex at O. Let P be a point on the parabola and A be a point on the x-axis such that $\angle OPA = 90^\circ$. Then the locus of the centroid of such triangles OPA is :

(1) $y^2 - 6x + 4 = 0$ (2) $y^2 - 9x + 6 = 0$
 (3) $y^2 - 2x + 8 = 0$ (4) $y^2 - 4x + 8 = 0$

Ans. (3)

Sol.



$m_{AP} = \frac{-t}{2}$

Equation of AP is

$y - 6t = \frac{-t}{2}(x - 3t^2)$

Put $y = 0 \Rightarrow x = 12 + 3t^2$

$\Rightarrow A(12 + 3t^2, 0)$

Let centroid of ΔOPA be $G(h, k)$

$\Rightarrow 3h = 0 + 3t^2 + 12 + 3t^2$

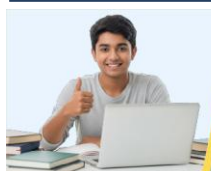
$3k = 0 + 6t + 0$

$\Rightarrow t = \frac{k}{2}, h = 2t^2 + 4$

$\Rightarrow h = 2\frac{k^2}{4} + 4$

$\Rightarrow$ Locus of (h, k) is

$y^2 = 2x - 8$



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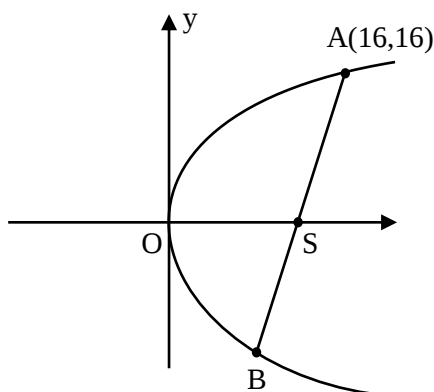
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6. Let one end of a focal chord of the parabola $y^2 = 16x$ be $(16, 16)$. If $P(\alpha, \beta)$ divides this focal chord internally in the ratio $5 : 2$, then the minimum value of $\alpha + \beta$ is equal to :

- (1) 22 (2) 7
(3) 5 (4) 16

Ans. (2)

Sol. $y^2 = 16x$

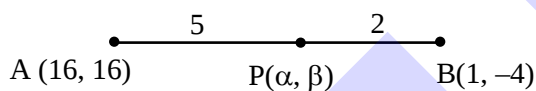


$\therefore$ parameter of point A is $t = 2$

$\Rightarrow$ Parameter of point B is $t = -\frac{1}{2}$

$\Rightarrow$ Coordinates of B is $(1, -4)$

Case 1 :

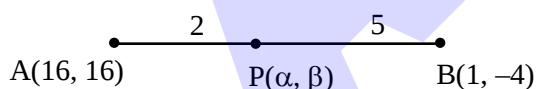


$$\alpha = \frac{5+32}{7} = \frac{37}{7}$$

$$\beta = \frac{-20+32}{7} = \frac{12}{7}$$

$$\Rightarrow \alpha + \beta = 7$$

Case 2 :



$$\alpha = \frac{2+80}{7},$$

$$\beta = \frac{-8+80}{7}$$

$$\alpha + \beta = 22$$

So minimum value of $\alpha + \beta = 7$

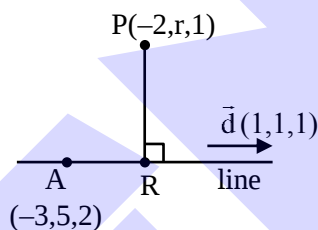
7. Let the line L pass through the point $(-3, 5, 2)$ and make equal angles with the positive coordinate axes. If the distance of L from the point $(-2, r, 1)$ is $\sqrt{\frac{14}{3}}$, then the sum of all possible values of r is :

- (1) 12 (2) 16
(3) 6 (4) 10

Ans. (4)

Sol. Equation line is : $\frac{x+3}{1} = \frac{y-5}{1} = \frac{z-2}{1} = \lambda$

$\therefore$ General point R on line is $R(\lambda-3, \lambda+5, \lambda+2)$



$$\vec{PR} = (\lambda-1, \lambda+5-r, \lambda+1)$$

$$\text{Now } \vec{PR} \cdot \vec{d} = 0$$

$$\Rightarrow (\lambda-1)1 + (\lambda+5-r)1 + (\lambda+1)1 = 0$$

$$\Rightarrow 3\lambda - r + 5 = 0$$

$$\Rightarrow \lambda = \frac{r-5}{3}$$

$$\therefore R = \left(\frac{r-5}{3} - 3, \frac{r-5}{3} + 5, \frac{r-5}{3} + 2 \right)$$

$$R = \left(\frac{r-14}{3}, \frac{r+10}{3}, \frac{r+1}{3} \right)$$

Now

$$PR = \sqrt{\frac{14}{3}} \Rightarrow (PR)^2 = \frac{14}{3}$$

$$\Rightarrow \left(\frac{r-14}{3} + 2 \right)^2 + \left(\frac{r+10}{3} - r \right)^2 + \left(\frac{r+1}{3} - 1 \right)^2 = \frac{14}{3}$$

$$\Rightarrow \frac{(r-8)^2}{9} + \frac{(10-2r)^2}{9} + \frac{(r-2)^2}{9} = \frac{14}{3}$$

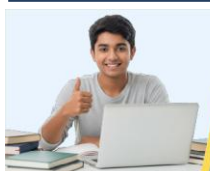
$$\Rightarrow (r^2 - 16r + 64) + (100 + 4r^2 - 40r) + (r^2 - 4r + 4) = 42$$

$$\Rightarrow 6r^2 - 60r + 126 = 0$$

$$\Rightarrow r^2 - 10r + 21 = 0$$

$$\Rightarrow r = 3, 7$$

sum of possible value of r is = 10



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8. Let the line L_1 be parallel to the vector $-3\hat{i} + 2\hat{j} + 4\hat{k}$ and pass through the point $(2, 6, 7)$ and the line L_2 be parallel to the vector $2\hat{i} + \hat{j} + 3\hat{k}$ and pass through the point $(4, 3, 5)$. If the line L_3 is parallel to the vector $-3\hat{i} + 5\hat{j} + 16\hat{k}$ and intersects the lines L_1 and L_2 at the points C and D, respectively, then $|\overline{CD}|^2$ is equal to :

- (1) 171 (2) 290
(3) 312 (4) 89

Ans. (2)

Sol. $L_1: \frac{x-2}{-3} = \frac{y-6}{2} = \frac{z-7}{4}$

Point C on $L_1: (-3\lambda_1 + 2, 2\lambda_1 + 6, 4\lambda_1 + 7)$

$L_2: \frac{x-4}{2} = \frac{y-3}{1} = \frac{z-5}{3}$

Point D on $L_2: (2\lambda_2 + 4, \lambda_2 + 3, 3\lambda_2 + 5)$

Dir's of line L_3 :

$L_3: \frac{2\lambda_2 + 3\lambda_1 + 2}{-3} = \frac{\lambda_2 - 2\lambda_1 - 3}{5} = \frac{3\lambda_2 - 4\lambda_1 - 2}{16}$

$\lambda_1 = -3, \lambda_2 = 2$

C (11, 0, -5)

D (8, 5, 11)

$|\overline{CD}|^2 = 3^2 + 5^2 + 16^2 = 290$

9. Let α and β be the roots of equation $x^2 + 2ax + (3a + 10) = 0$ such that $\alpha < 1 < \beta$. Then the set of all possible values of a is :

- (1) $\left(-\infty, \frac{-11}{5}\right) \cup (5, \infty)$ (2) $(-\infty, -2) \cup (5, \infty)$
(3) $(-\infty, -3)$ (4) $\left(-\infty, \frac{-11}{5}\right)$

Ans. (4)

Sol. $\because \alpha < 1 < \beta$

$f(1) < 0$

$\Rightarrow 1 + 2a + (3a + 10) < 0$

$\Rightarrow 5a + 11 < 0$

$a < \frac{-11}{5}$

$\therefore a \in \left(-\infty, \frac{-11}{5}\right)$

10. A random variable X takes values 0, 1, 2, 3 with probabilities $\frac{2a+1}{30}, \frac{8a-1}{30}, \frac{4a+1}{30}, b$ respectively,

where $a, b \in \mathbf{R}$. Let μ and σ respectively be the mean and standard deviation of X such that $\sigma^2 + \mu^2 = 2$.

Then $\frac{a}{b}$ is equal to :

- (1) 30 (2) 3
(3) 60 (4) 12

Ans. (3)

x	0	1	2	3
p(x)	$\frac{2a+1}{30}$	$\frac{8a-1}{30}$	$\frac{4a+1}{30}$	b

Sol.

$\sigma^2 = \sum x_i^2 p(x_i) - \mu^2$

$\sigma^2 + \mu^2 = \sum x_i^2 p(x_i)$

$= 0 + 1 \left(\frac{8a-1}{30} \right) + 4 \left(\frac{4a+1}{30} \right) + 9b$

$\Rightarrow \frac{24a + 270b + 3}{30} = 2$

$24a + 270b = 57$

$8a + 90b = 19 \dots (1)$

Also

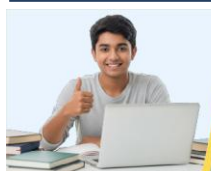
$\sum p(i) = 1$

$\frac{2a+1}{30} + \frac{8a-1}{30} + \frac{4a+1}{30} + b = 1$

$14a + 30b = 29 \dots (2)$

Solving (1) & (2)

$a = 2, b = \frac{1}{30}, \frac{a}{b} = 60$



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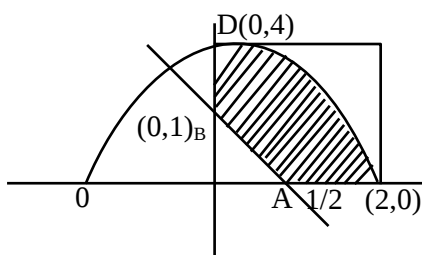
11. If the area of the region $\{(x, y) : 1 - 2x \leq y \leq 4 - x^2, x \geq 0, y \geq 0\}$ is $\frac{\alpha}{\beta}$, $\alpha, \beta \in \mathbb{N}$, $\gcd(\alpha, \beta) = 1$, then the

value of $(\alpha + \beta)$ is :

- (1) 73 (2) 85
(3) 91 (4) 67

Ans. (1)

Sol.



$$\text{Required area} = \frac{2}{3} \times 8 - \frac{1}{2} \times \frac{1}{2} \times 1$$

$$= \frac{16}{3} - \frac{1}{4} = \frac{61}{12} = \frac{\alpha}{\beta}$$

$$\Rightarrow \alpha + \beta = 73$$

12. Let $a_1, \frac{a_2}{2}, \frac{a_3}{2^2}, \dots, \frac{a_{10}}{2^9}$ be a G.P. of common ratio

$\frac{1}{\sqrt{2}}$. If $a_1 + a_2 + \dots + a_{10} = 62$, then a_1 is equal to :

- (1) $2(\sqrt{2} - 1)$ (2) $2 - \sqrt{2}$
(3) $\sqrt{2} - 1$ (4) $2(2 - \sqrt{2})$

Ans. (1)

Sol. $\frac{a_2}{2a_1} = \frac{a_3}{2a_2} = \frac{a_4}{2a_3} = \dots = \frac{a_{10}}{2a_9} = \frac{1}{\sqrt{2}}$

$\therefore a_1, a_2, a_3, \dots, a_{10}$ are in G.P. with common ratio $\sqrt{2}$.

$$\sum_{i=1}^{10} a_i = \frac{a_1((\sqrt{2})^{10} - 1)}{\sqrt{2} - 1} = 62$$

$$\Rightarrow a_1 = 2(\sqrt{2} - 1)$$

13. Let $A = \{x : |x^2 - 10| \leq 6\}$ and $B = \{x : |x - 2| > 1\}$. Then

- (1) $A \cup B = (-\infty, 1] \cup (2, \infty)$
(2) $A - B = [2, 3]$
(3) $A \cap B = [-4, -2] \cup [3, 4]$
(4) $B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty)$

Ans. (4)

Sol. $|x^2 - 10| \leq 6$

$$-6 \leq x^2 - 10 \leq 6$$

$$4 \leq x^2 \leq 16$$

$$A = [-4, -2] \cup [2, 4]$$

$$|x - 2| > 1$$

$$B = (-\infty, 1) \cup (3, \infty)$$

$$A \cup B = (-\infty, 1) \cup [2, \infty)$$

$$A \cap B = [-4, -2] \cup (3, 4]$$

$$A - B = [2, 3]$$

$$B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty)$$

14. For the matrices $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -29 & 49 \\ -13 & 18 \end{bmatrix}$, if $(A^{15} + B) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then among the following which one is true?

- (1) $x = 5, y = 7$ (2) $x = 18, y = 11$
(3) $x = 11, y = 2$ (4) $x = 16, y = 3$

Ans. (3)

Sol. Here $A^n = \begin{bmatrix} 2n+1 & -4n \\ n & -2n+1 \end{bmatrix}$

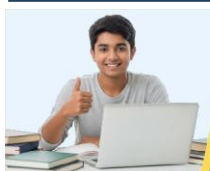
$$\Rightarrow A^{15} = \begin{bmatrix} 31 & -60 \\ 15 & -29 \end{bmatrix}$$

$$\Rightarrow A^{15} + B = \begin{bmatrix} 2 & -11 \\ 2 & -11 \end{bmatrix}$$

$$\text{Now } (A^{15} + B) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -11 \\ 2 & -11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 2x - 11y = 0$$



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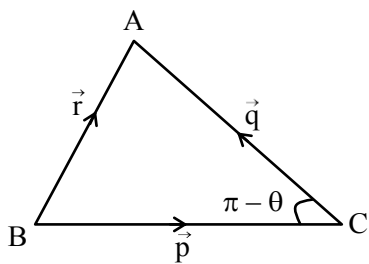
15. For a triangle ABC, let $\vec{p} = \overrightarrow{BC}$, $\vec{q} = \overrightarrow{CA}$ and $\vec{r} = \overrightarrow{BA}$. If $|\vec{p}| = 2\sqrt{3}$, $|\vec{q}| = 2$ and $\cos\theta = \frac{1}{\sqrt{3}}$,

where θ is the angle between $\vec{p}$ and $\vec{q}$, then

$|\vec{p} \times (\vec{q} - 3\vec{r})|^2 + 3|\vec{r}|^2$ is equal to:

- (1) 340 (2) 220
(3) 410 (4) 200

Ans. (4)



Sol.

$$\vec{p} + \vec{q} = \vec{r}$$

$$\cos(\pi - \theta) = \frac{|\vec{p}|^2 + |\vec{q}|^2 - |\vec{r}|^2}{2|\vec{p}||\vec{q}|}$$

$$\frac{-1}{\sqrt{3}} = \frac{12 + 4 - |\vec{r}|^2}{2 \cdot 2\sqrt{3} \cdot 2}$$

$$|\vec{r}|^2 = 24$$

$$\therefore |\vec{p} \times (\vec{q} - 3\vec{r})|^2 + 3|\vec{r}|^2$$

$$= |\vec{p} \times (\vec{q} - 3\vec{p} - 3\vec{q})|^2 + 72$$

$$= |\vec{p} \times (-3\vec{p} - 2\vec{q})|^2 + 72$$

$$= |-2\vec{p} \times \vec{q}|^2 + 72$$

$$= 4|\vec{p}|^2|\vec{q}|^2 \sin^2\theta + 72$$

$$= 4 \cdot 12 \cdot 4 \cdot \frac{2}{3} + 72$$

$$= 200$$

16. Let $y = y(x)$ be the solution of the differential equation $\sec x \frac{dy}{dx} - 2y = 2 + 3\sin x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

$y(0) = -\frac{7}{4}$. Then $y\left(\frac{\pi}{6}\right)$ is equal to:

- (1) $-\frac{5}{2}$ (2) $-\frac{5}{4}$
(3) $-3\sqrt{3} - 7$ (4) $-3\sqrt{2} - 7$

Ans. (1)

Sol. $\frac{dy}{dx} - 2y \cos x = 2 \cos x + 3 \sin x \cdot \cos x$

$$\text{I.F.} = e^{-2\sin x}$$

$$e^{-2\sin x} \cdot y = \int e^{-2\sin x} (3 \sin x \cos x + 2 \cos x) dx$$

$$y \cdot e^{-2\sin x} = e^{-2\sin x} \left(-\frac{3}{2} \sin x - \frac{7}{4} \right) + C$$

$$\Rightarrow y = -\frac{3}{2} \sin x - \frac{7}{4} + C e^{2\sin x}$$

$$\because y(0) = -\frac{7}{4} \Rightarrow C = 0$$

$$y\left(\frac{\pi}{6}\right) = -\frac{3}{2} \cdot \frac{1}{2} - \frac{7}{4} = -\frac{5}{2}$$

17. Let $A = \{2, 3, 5, 7, 9\}$. Let R be the relation on A defined by $x R y$ if and only if $2x \leq 3y$. Let ℓ be the number of elements in R , and m be the minimum number of elements required to be added in R to make it a symmetric relation. Then $\ell + m$ is equal to:

- (1) 23 (2) 25
(3) 21 (4) 27

Ans. (2)

Sol. $A = \{2, 3, 5, 7, 9\}$

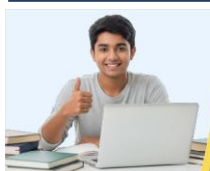
$$y \geq \frac{2x}{3}$$

$$\left. \begin{array}{l} x=2, y=2,3,5,7,9 \\ x=3, y=2,3,5,7,9 \\ x=5, y=5,7,9 \\ x=7, y=5,7,9 \\ x=9, y=7,9 \end{array} \right\} \rightarrow \ell = 18$$

to make it symmetric elements to be added are $\{(5,2), (7,2), (9,2), (5,3), (7,3), (9,3), (9,5)\}$

$$m = 7$$

$$\therefore \ell + m = 25$$



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18. If the system of equations

$$3x + y + 4z = 3$$

$$2x + \alpha y - z = -3$$

$$x + 2y + z = 4$$

has no solution, then the value of α is equal to:

- (1) 19 (2) 4
(3) 13 (4) 23

Ans. (1)

Sol. for no solution $\Delta = 0$

$$\begin{vmatrix} 3 & 1 & 4 \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 3(\alpha + 2) + 1(-1 - 2) + 4(4 - \alpha) = 0$$

$$\Rightarrow 19 - \alpha = 0 \Rightarrow \boxed{\alpha = 19}$$

& for $\alpha = 19$

$$\Delta_x = \begin{vmatrix} 3 & 1 & 4 \\ -3 & 19 & -1 \\ 4 & 2 & 1 \end{vmatrix} = 3(21) + 1(-1) + 4(-82)$$

$\neq 0$

$\therefore$ no solution for $\alpha = 19$

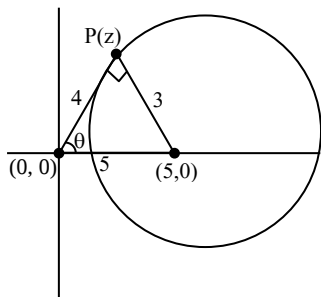
19. Let z be the complex number satisfying $|z - 5| \leq 3$ and having maximum positive principal argument.

Then $34 \left| \frac{5z - 12}{5iz + 16} \right|^2$ is equal to:

- (1) 16 (2) 12
(3) 26 (4) 20

Ans. (4)

Sol.



$$|z - 5| \leq 3$$

For $\arg(z)$ to be maximum, z lies at P.

$$z = (4 \cos \theta, 4 \sin \theta)$$

$$\equiv \left(4 \cdot \left(\frac{4}{5} \right), 4 \cdot \left(\frac{3}{5} \right) \right) = \left(\frac{16}{5}, \frac{12}{5} \right) = \frac{16}{5} + \frac{12i}{5}$$

$$\text{Now, } 34 \left| \frac{5z - 12}{5iz + 16} \right|^2 = 34 \left| \frac{(16 + 12i) - 12}{(16i - 12) + 16} \right|^2$$

$$= 34 \left| \frac{4 + 12i}{16i + 4} \right|^2$$

$$= 34 \left(\frac{16 + 144}{256 + 16} \right) = 34 \left(\frac{160}{272} \right) = 20$$

20. The largest $n \in \mathbb{N}$, for which 7^n divides $101!$, is :

- (1) 16 (2) 18
(3) 15 (4) 19

Ans. (1)

Sol. Exponent of 7 in $101!$

$$= \left[\frac{101}{7} \right] + \left[\frac{101}{7^2} \right] + \left[\frac{101}{7^3} \right] + \dots$$

$$= 14 + 2 = 16$$

SECTION-B

21. Let $[\cdot]$ denote the greatest integer function and

$f(x) = \lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{k=1}^n \left[\frac{k^2}{3^x} \right]$. Then $12 \sum_{j=1}^{\infty} f(j)$ is equal to _____.

Ans. (2)

$$\text{Sol. } \sum_{k=1}^n \left(\frac{k^2}{3^x} - 1 \right) < \sum_{k=1}^n \left[\frac{k^2}{3^x} \right] \leq \sum_{k=1}^n \frac{k^2}{3^x}$$

$$\frac{n(n+1)(2n+1)}{6 \cdot 3^x} < \sum_{k=1}^n \left[\frac{k^2}{3^x} \right] \leq \frac{n(n+1)(2n+1)}{6 \cdot 3^x}$$

$$\lim_{n \rightarrow \infty} \frac{n(n+1)(2n+1)}{6n^3 \cdot 3^x} < \lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{k=1}^n \left[\frac{k^2}{3^x} \right] \leq \lim_{n \rightarrow \infty} \frac{n(n+1)(2n+1)}{6 \cdot 3^x \cdot n^3}$$

$$\frac{1}{3^{x+1}} < \lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{k=1}^n \left[\frac{k^2}{3^x} \right] \leq \frac{1}{3^{x+1}}$$

$$\Rightarrow f(x) = \frac{1}{3^{x+1}}$$

$$\Rightarrow 12 \sum_{j=1}^{\infty} f(j) = 12 \sum_{j=1}^{\infty} \frac{1}{3^{j+1}} = 12 \left[\frac{1}{9} + \frac{1}{27} + \dots \right]$$

$$= 12 \cdot \left(\frac{\frac{1}{9}}{1 - \frac{1}{3}} \right) = 2$$



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22. If $\int_0^1 4 \cot^{-1}(1-2x+4x^2) dx = a \tan^{-1}(2) - \log_e(5)$, where $a, b \in \mathbb{N}$, then $(2a+b)$ is equal to _____.

Ans. (9)

Sol. Let $I = \int_0^1 \cot^{-1}(1-2x+4x^2) dx$

$$I = \int_0^1 (\cot^{-1}(2x-1) - \cot^{-1}(2x)) dx \quad \dots(1)$$

Applying king

$$I = \int_0^1 (-\cot^{-1}(2x-1) + \cot^{-1}(2x-2)) dx \quad \dots(2)$$

From (1) & (2)

$$2I = \int_0^1 (\cot^{-1}(2x-2) - \cot^{-1}(2x)) dx$$

$$= \int_0^1 \cot^{-1}(2x-2) dx - \int_0^1 \cot^{-1}(2x) dx$$

Applying King

$$= \int_0^1 \cot^{-1}(-2x) dx - \int_0^1 \cot^{-1}(2x) dx$$

$$= \int_0^1 (\pi - \cot^{-1}(2x)) dx - \int_0^1 \cot^{-1}(2x) dx$$

$$= \int_0^1 (\pi - 2 \cot^{-1}(2x)) dx$$

$$= \pi - 2 \int_0^1 (\cot^{-1} 2x) \cdot 1 dx$$

By parts

$$= \pi - 2 \left[(x \cot^{-1} 2x)_0^1 + \int_0^1 \frac{2x}{1+4x^2} dx \right]$$

Let $1+4x^2 = t$
 $8x dx = dt$

$$= \pi - 2 \left[\cot^{-1} 2 + \frac{1}{4} \int_1^5 \frac{dt}{t} \right]$$

$$= \pi - 2 \cot^{-1} 2 - \frac{1}{2} \ln 5$$

$$2I = 2 \tan^{-1} 2 - \frac{1}{2} \ln 5$$

$$\Rightarrow 4I = 4 \tan^{-1} 2 - \ln 5$$

$$\therefore 2a + b = 8 + 1 = 9$$

23. Let the maximum value of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ for $x \in \left[-\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right]$ be $\frac{m}{n} \pi^2$, where $\gcd(m, n) = 1$.

Then $m+n$ is equal to _____.

Ans. (65)

Sol. $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$

$$= (\sin^{-1}x + \cos^{-1}x)^2 - 2 \sin^{-1}x \cos^{-1}x$$

$$= \frac{\pi^2}{4} - 2 \left(\sin^{-1}x \right) \left(\frac{\pi}{2} - \sin^{-1}x \right)$$

$$= 2 \left(\sin^{-1}x - \frac{\pi}{4} \right)^2 + \frac{\pi^2}{8} \quad \text{where } \sin^{-1}x \in \left[-\frac{\pi}{3}, \frac{\pi}{4} \right]$$

Then max value occurs at $\sin^{-1}x = -\frac{\pi}{3}$

$$\text{Which is } 2 \left(\frac{\pi}{3} + \frac{\pi}{4} \right)^2 + \frac{\pi^2}{8} = \frac{29\pi^2}{36}$$

$$\Rightarrow m = 29 \text{ and } n = 36$$

$$\therefore m+n = 65$$

24. If

$$\left(\frac{1}{{}^{15}C_0} + \frac{1}{{}^{15}C_1} \right) \left(\frac{1}{{}^{15}C_1} + \frac{1}{{}^{15}C_2} \right) \dots \left(\frac{1}{{}^{15}C_{12}} + \frac{1}{{}^{15}C_{13}} \right) = \frac{\alpha^{13}}{{}^{14}C_0 \cdot {}^{14}C_1 \dots {}^{14}C_{12}}, \text{ then } 30\alpha \text{ is equal to } \underline{\hspace{2cm}}.$$

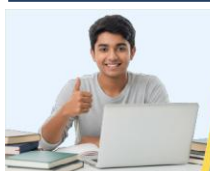
Ans. (32)

$$\text{Sol. } \prod_{r=0}^{12} \left(\frac{1}{{}^{15}C_r} + \frac{1}{{}^{15}C_{r+1}} \right) = \prod_{r=0}^{12} \frac{\frac{16}{r+1} \cdot {}^{15}C_r}{{}^{15}C_r \cdot {}^{15}C_{r+1}}$$

$$= \prod_{r=0}^{12} \frac{16}{(r+1) \cdot \frac{15}{r+1} \cdot {}^{14}C_r} = \prod_{r=0}^{12} \frac{\left(\frac{16}{15} \right)}{{}^{14}C_r}$$

$$= \frac{\left(\frac{16}{15} \right)^{13}}{{}^{14}C_0 \cdot {}^{14}C_1 \dots {}^{14}C_{12}} \Rightarrow \alpha = \frac{16}{15}$$

$$\Rightarrow 30\alpha = 32$$



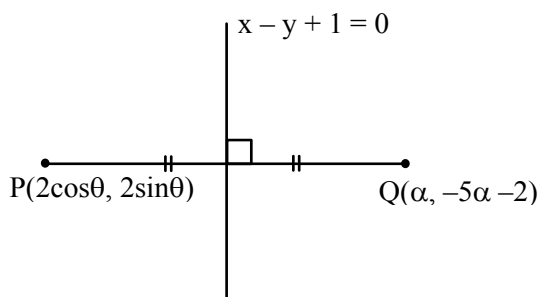
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25. If P is a point on the circle $x^2 + y^2 = 4$, Q is a point on the straight line $5x + y + 2 = 0$ and $x - y + 1 = 0$ is the perpendicular bisector of PQ, then 13 times the sum of abscissa of all such point P is _____.

Ans. (2)



Mid point of PQ lies on $x - y + 1 = 0$

$$\frac{2\cos\theta + \alpha}{2} - \frac{2\sin\theta - 5\alpha - 2}{2} + 1 = 0$$

$$2\cos\theta + \alpha - 2\sin\theta + 5\alpha + 2 + 2 = 0$$

$$\cos\theta - \sin\theta + 3\alpha + 2 = 0 \quad \dots(1)$$

$\therefore$ Slope of PQ is -1

$$\frac{2\sin\theta + 5\alpha + 2}{2\cos\theta - \alpha} = -1$$

$$2\sin\theta + 5\alpha + 2 = -2\cos\theta + \alpha$$

$$\sin\theta + \cos\theta + 2\alpha + 1 = 0 \quad \dots(2)$$

eliminate α from (1) and (2)

$$\Rightarrow \cos\theta + 5\sin\theta = 1, \theta \in [0, 2\pi]$$

$$\Rightarrow 5 \times 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2\sin^2\frac{\theta}{2}$$

$$\therefore \sin\frac{\theta}{2} = 0 \Rightarrow \cos\theta = 1$$

or

$$\sin\frac{\theta}{2} = 5 \Rightarrow \cos\theta = -\frac{12}{13}$$

Sum of all possible values of abscissa of point P is

$$= 2 \times 1 + 2 \left(-\frac{12}{13} \right) = \frac{2}{13}$$

$\therefore$ 13 times sum of all possible values of abscissa of point P is 2.



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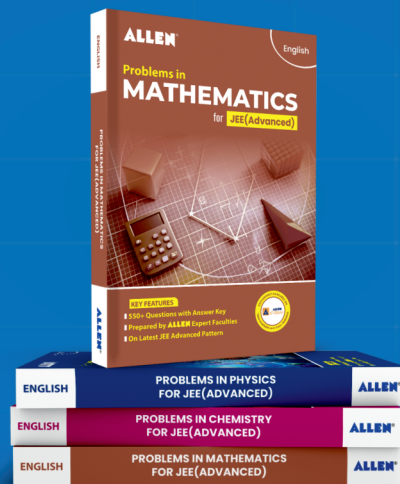
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